AIAA 81-4326

Nongradient Theory for Oblique Turbulent Flames with Premixed Reactants

Goro Masuya* and Paul A. Libby†
University of California, San Diego, La Jolla, Calif.

An earlier analysis based on second-order closure and the Bray-Moss-Libby model of premixed combustion is applied to infinite, planar turbulent flames which are oblique to the oncoming reactants and which have undeflected mean streamlines. In such flames all three mechanisms, dilatation, Reynolds shear stresses, and mean pressure gradient, contribute to the balance of turbulent kinetic energy. Examination of the appropriate conservation equations in primitive form indicates that the intensity of the fluctuations of the velocity component normal to the flame and the mean flux of product in that direction are independent of obliquity. Thus earlier findings regarding countergradient diffusion and production of turbulence due to the mean pressure gradient prevail in oblique flames. The mean flux of product in the tangential direction and the intensity of the fluctuations of the velocity component in that direction are calculated. It is found that the mean streamlines of parcels of reactants and products are significantly different from one another and from the mean streamline. The intensities downstream of highly oblique turbulent flames are predicted to lead to two-dimensional turbulence in the plane containing the normal and tangential coordinates.

Nomenclature

B_u, C_u, D_v	= coefficients in expansions about $\tilde{c}=1$, Eqs. (34)
c	= progress variable or normalized product mass fraction
$F_u, F_v, \hat{F}_v$	= nondimensional parameters yielding the mean flux of product, Eqs. (25)
f	= interior, $0<c<1$, pdf distribution
h	$=(1+\tau\tilde{c})/[\tilde{c}(1-\tilde{c})]$
I_u, I_v	= nondimensional intensity parameters, Eqs. (25)
$K_u, K_v, L_{uu}, L_{vv}, \ldots$	= nondimensional mean flux parameters, Eqs. (25)
$P(c,x), P(u,c,x), \ldots$	= probability density functions
p	= static pressure
T	= temperature
u, v, w	= velocity components in the x,y,z-coordinate directions
$\bar{\dot{w}}$	= mean rate of production of product
x-, y-, z	= Cartesian coordinate
α, β, γ	= strengths of three modes of product concentration
$\beta_{u0}, \gamma_{u0}, \ldots$	= coefficients in expansions about $\tilde{c}=0$, Eqs. (33)
$\beta_{u1}, \gamma_{u1}, \ldots$	= coefficients in expansions about $\tilde{c}=1$, Eqs. (34)
ρ	= mass density
$\kappa_{uu}, \kappa_{uc}, \ldots$	= empirical constants
θ	= turbulent flame angle
τ	= heat release parameter
$\hat{\tau}$	$=\tau/\tan\theta$, heat release-flame angle parameter
λ_u, λ_v	= empirical constants
$\chi_{uu}, \chi_{uc}, \ldots$	= dissipation terms
ψ_0, ψ_r, ψ_p	= flow deflections

Subscripts

0	= conditions upstream of reaction zone
∞	= conditions downstream of reaction zone
r,p	= conditions in reactants and products

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*Research Associate, Department of Applied Mechanics and Engineering Sciences. Permanent address: National Aerospace Laboratory, Kakuda, Miyagi, Japan.

†Professor of Fluid Mechanics, Department of Applied Mechanics and Engineering Sciences. Fellow AIAA.

I. Introduction

ONE of the most interesting aspects of turbulent reacting flows relates to the influence on the turbulence of the heat release accompanying chemical reaction in many cases of practical interest. Bray[1] shows that the balance equation of turbulent kinetic energy involves a competition between the diminution in such energy due to dilatation and its creation due to Reynolds stresses. In making his considerations Bray follows common practice and neglects the effect of mean pressure gradients. The usual argument justifying such neglect is that the pressure drop across the reaction zone in which the heat release and thus the influence of interest occur is small, being proportional to the square of the Mach number based on the flame speed. We now know from recent work that this argument does indeed apply to the thermochemical behavior of the fluid but that dynamic coupling between the small mean pressure drop and the large density inhomogeneities arising in flames of practical interest leads to significant fluid mechanical effects, e.g., countergradient diffusion[2] and the production of turbulent kinetic energy.[3] Thus we recognize that the mean pressure gradient across the reaction zone introduces a third mechanism in the balance of turbulent kinetic energy.

References 2 and 3 are concerned only with either normal flames or their equivalent, oblique flames with unconstrained mean streamlines such that the mean velocity tangent to the flame is unaltered within the reaction zone. In such flames there are no Reynolds stresses so that only dilatation and mean pressure gradient alter the turbulent kinetic energy. In the present paper we extend earlier work and analyze premixed turbulent flames with constrained mean streamlines. We thus deal with flames involving all three mechanisms influencing the balance of turbulent kinetic energy.

The flames we consider are shown schematically in Fig. 1; involved are infinite planar reaction zones inclined at a specified angle θ to the oncoming reactants. To emphasize the

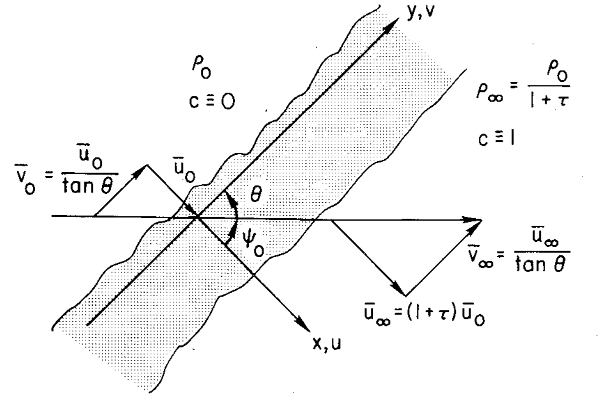

Fig. 1 Schematic representation of an oblique planar flame.

influence of Reynolds stresses on the characteristics of premixed flames we consider the limiting case of an oblique flame in which the mean streamline is undeflected and which represents an idealization of flames confined within a duct.[4] The actual streamlines in such flows are determined by global hydrodynamic considerations and, in general, are deflected at small angles with respect to the duct wall both upstream and downstream of the flame. Thus we view the flames treated here and shown in Fig. 1 in terms of a limiting behavior, behavior opposite to that associated with flames having unconstrained streamlines. Most flames of practical interest are expected to exhibit characteristics intermediate to these two limits.

The chemical system we consider is that described by the Bray-Moss model of premixed combustion.[5] Accordingly, the thermochemical state is completely described by a variable $c(x,t)$ which may be considered a progress variable with the value zero in reactants, the value unity in products, and intermediate values in mixture undergoing chemical reaction. It may also, as we prefer, be considered the product mass fraction normalized to have the value unity in fully burned mixture. Thus the probability density function $P(c;x)$ is represented by delta functions with strengths $\alpha(x)$ and $\beta(x)$ at $c=0$, 1, respectively, and by the interior distribution $\gamma(x)$ $f(c;x)$. With the integral of f normalized to unity we have $\alpha(x)+\beta(x)+\gamma(x)=1$. We adopt the assumption of fast chemistry, namely, that $\gamma \ll 1$. Thus $P(c;x)$ is effectively bimodal. A further consequence of γ small is that $\alpha(x)=[1-\tilde{c}(x)]/[1+\tau\tilde{c}(x)]$ and $\beta(x)=(1+\tau)\tilde{c}(x)/[1+\tau\tilde{c}(x)]$ where we use Favre averaging so that $\tilde{c}=\overline{\rho c}/\bar{\rho}$ and where τ is a heat release parameter. Thus we see that the strengths of the delta functions are given in a simple fashion by the mean concentration $\tilde{c}(x)$.

Bimodality implies that the function $c(t;x)$ is a conditioning function analogous to the intermittency function used to discriminate turbulent and irrotational fluid. From this point of view the joint velocity-state pdf's are seen to be dominated by conditioned pdf's corresponding to velocity distributions when $c=0$, 1. Thus, for example, $P(u,c;x)=\alpha(x)P(u,0;x)+\beta(x)P(u,1;x)+O(\gamma)$. The consideration of conditioned pdf's is useful relative to both experiments in premixed flames (see Moss[6] and Shephard and Moss[7]) and to guide the modeling required to achieve closure in theoretical studies.

Bimodality implies a further important feature of the analysis, namely, that details of the interior distribution $f(c;x)$ are inessential. Thus, it is unnecessary either to adopt an explicit mode of chemical reaction at the molecular level or to make assumptions about the various chemical and turbulence scales operative. However, we note that in the present and related studies we have estimated $f(c;x)$ on the basis of a laminar flamelet model of reacting surfaces because we believe it to be realistic in many circumstances. Such estimates do not restrict the generality of the results.

In terms of the variable c the density and temperature are given by

$$\rho=\rho_0/(1+\tau c), \quad T=T_0(1+\tau c)$$

where the subscript 0 denotes conditions in the oncoming reactants; from these equations $\tau=[(T_\infty/T_0)-1]$ where the subscript ∞ denotes conditions downstream of the reaction zone. The parameter τ plays an important role in the analysis, describes the extent of the heat release, and is simply related to physically significant and readily measured quantities.

In Refs. 2 and 3 this thermochemical system is incorporated in a theory for infinite, planar normal flames based on second-order closure and avoiding gradient transport throughout. It is thus possible to evaluate the validity in premixed turbulent flames of the ubiquitous gradient transport model; it is found that for small degrees of heat release, i.e., for $\tau<1$, gradient transport is operative but that for values of τ of practical interest the mean flux of product $\overline{\rho u''c''}>0$ throughout most of the reaction zone, indicating that gradient transport yields even the incorrect sign of this flux. The explanation for this failure resides in the differential effect on parcels of heavy reactants and light products of the small mean pressure drop across the flame. Such an effect tends to drive the parcels of product toward the region of high mean product concentration contrary to gradient transport. Furthermore, with $\overline{\rho u''c''}>0$ the mean pressure gradient leads to turbulence production so that for values of $\tau>4$ the turbulence downstream of the reaction zone exceeds that in the oncoming reactants. Thus, dilatation which diminishes turbulent kinetic energy is overwhelmed by the effect of the mean pressure gradient. In Ref. 3 the theory is shown to be in "encouraging" agreement with the recent data of Moss[6] appropriately interpreted.

In extending the analysis of Refs. 2 and 3 to oblique flames many of the details carry over without alteration and, thus, in the interests of brevity can be presented without discussion. This permits us to emphasize the new aspects of the analysis required by obliquity.

It is unfortunate that no detailed measurements relative to oblique flames suitable for comparison with the present theory are available. However, we shall see that the intensity of the fluctuations of the velocity component normal to the flame and the mean flux of product in that direction, quantities shown in Ref. 3 to be in agreement with experiment, are unaltered by obliquity. Thus we have limited but important support for the present theory. With regard to experiment our predictions relative to the two sets of conditioned velocities, i.e., the mean velocities within reactants and products in the x- and y-coordinate directions, suggest important measurements which can be carried out without difficulty by the diagnostic method used by Moss,[6] Shephard and Moss,[7] and Kennedy and Kent.[8]

II. Analysis

The following Favre-averaged conservation equations are used in the present analysis.

Conservation of mass

$$\frac{d}{dx}(\bar{\rho}\tilde{u})=0 \tag{1}$$

Conservation of x momentum

$$\frac{d}{dx}(\bar{\rho}\tilde{u}^2+\overline{\rho u''^2})=-\frac{d\bar{p}}{dx} \tag{2}$$

Conservation of y momentum

$$\frac{d}{dx}(\bar{\rho}\tilde{u}\tilde{v}+\overline{\rho u''v''})=0 \tag{3}$$

Conservation of product

$$\frac{\mathrm{d}}{\mathrm{d}x}(\bar{\rho}\tilde{u}\tilde{c}+\overline{\rho u''c''})=\bar{\omega} \tag{4}$$

Conservation of Reynolds stresses

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\bar{\rho}\tilde{u}\frac{\overline{\rho u''^2}}{\bar{\rho}}+\overline{\rho u''^3}\right]+2\overline{\rho u''^2}\frac{\mathrm{d}\tilde{u}}{\mathrm{d}x}=-2\overline{u''}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\bar{\chi}_{uu} \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\bar{\rho}\tilde{u}\frac{\overline{\rho v''^2}}{\bar{\rho}}+\overline{\rho u''v''^2}\right]+2\overline{\rho u''v''}\frac{\mathrm{d}\tilde{v}}{\mathrm{d}x}=-\bar{\chi}_{vv} \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\bar{\rho}\tilde{u}\frac{\overline{\rho w''^2}}{\bar{\rho}}+\overline{\rho u''w''^2}\right]=0 \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\bar{\rho}\tilde{u}\frac{\overline{\rho u''v''}}{\bar{\rho}}+\overline{\rho u''^2v''}\right]+\overline{\rho u''v''}\frac{\mathrm{d}\tilde{u}}{\mathrm{d}x}+\overline{\rho u''^2}\frac{\mathrm{d}\tilde{v}}{\mathrm{d}x}$$
$$=-\overline{v''}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\bar{\chi}_{uv} \tag{8}$$

Conservation of flux of product in the x direction

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\bar{\rho}\tilde{u}\frac{\overline{\rho u''c''}}{\bar{\rho}}+\overline{\rho u''^2c''}\right]+\overline{\rho u''c''}\frac{\mathrm{d}\tilde{u}}{\mathrm{d}x}+\overline{\rho u''^2}\frac{\mathrm{d}\tilde{c}}{\mathrm{d}x}$$
$$=-\overline{c''}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}+\overline{u''\omega}-\bar{\chi}_{uc} \tag{9}$$

Conservation of flux of product in the y direction

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\bar{\rho}\tilde{u}\frac{\overline{\rho v''c''}}{\bar{\rho}}+\overline{\rho u''v''c''}\right]+\overline{\rho u''c''}\frac{\mathrm{d}\tilde{v}}{\mathrm{d}x}$$
$$+\overline{\rho u''v''}\frac{\mathrm{d}\tilde{c}}{\mathrm{d}x}=-\overline{v''\omega}-\bar{\chi}_{vc} \tag{10}$$

In these equations molecular effects are confined to the mean dissipation terms $\bar{\chi}_{uu}$, $\bar{\chi}_{vv}$, etc. While the effect of mean pressure gradient is included via Eq. (2), pressure fluctuations are neglected on the basis of the following argument. The pressure-velocity and pressure-rate-of-strain models developed over a period of years for constant density turbulence do not relate to pressure fluctuations due to combustion, i.e., due to volumetric changes, and are therefore irrelevant to the present analysis. If these effects are to be included, new models therefore are required. However, the main effect of pressure fluctuations, whether combustion-induced or otherwise, is expected to be a redistribution of the intensities of the velocity fluctuations in the three coordinate directions. Since the analysis of Refs. 2 and 3 indicates that high degree of anisotropy prevail in premixed flames, it might be argued that within flames such redistribution effects could be important; but both theory and experiment indicate that premixed flames are relatively thin, being on the order of the integral scale in the oncoming reactants in extent. Thus the flow length within the reaction zone available for redistribution to be effected is considered to be insufficient for pressure fluctuations to be significant. Of course, if an entire flowfield, involving turbulent reactants, a reaction zone such as we analyze, and turbulent products, is dealt with, there will be adequate flow length for redistribution of anisotropies generated within the reaction zone to take place and pressure fluctuations both combustion-induced and otherwise must be taken into account. This argument should be subjected to close examination and we welcome contributions to that end.

It is immediately evident that Eqs. (1), (2), (4), (5), (7), and (9) are independent of the angle of the flame θ. Moreover, examination indicates that the boundary conditions applicable to these equations are likewise independent of θ. Since these are the equations used in Refs. 2 and 3 to analyze normal flames, we conclude that the predicted distributions of $\bar{\rho}$, $\tilde{u}$, $\mathrm{d}\bar{p}/\mathrm{d}x$, $\bar{\omega}$, $\overline{\rho u''^2}$, $\overline{\rho u''c''}$, and the conditioned statistics of the u-velocity component are unaltered by obliquity.‡ Thus the processes of turbulent transport and turbulence production in normal flames described in Refs. 2 and 3 prevail in the normal direction in oblique flames as well. This finding permits attention to be focused on the remaining equations and on the variables they describe.

The assumption of a constrained mean streamline is stated by the requirement that

$$\tilde{v}=\tilde{u}/\tan\theta \tag{11}$$

Moreover, with Eq. (11) the Reynolds shear stress is given without approximation by integration of Eq. (3); under the assumption of zero shear stress in the oncoming reactants we have[9]

$$\frac{\overline{\rho u''v''}}{\rho_0\tilde{u}_0^2}=-\frac{\tau\tilde{c}}{\tan\theta}=-\hat{\tau}\tilde{c} \tag{12}$$

where the parameter $\hat{\tau}$ is obviously defined and is introduced for notational convenience. Note that the left side of Eq. (12) corresponds to a force coefficient with the value $\hat{\tau}$ when $\tilde{c}=1$. The variation of the Reynolds shear stress across the reaction zone given by Eq. (12) implies that there is a force field acting on the gas in the y direction; indeed it is this force which constrains the mean flow to be undeflected within the reaction zone.§

Equation (7) implies that $\overline{\rho w''^2}/\bar{\rho}=\text{const}=\overline{w'^2_0}$, i.e., the intensity of the velocity component tangent to the flame in the z direction is unaltered within the reaction zone. In normal flames a similar argument applies to $\overline{\rho v''^2}/\bar{\rho}$ so that in such flames isotropic turbulence in the oncoming reactants becomes cylindrically symmetric downstream of the reaction zone. In oblique flames only the intensity of the w-velocity component remains unchanged so that the most general anisotropy is predicted downstream of oblique flames.

Conditioned Probability Density Functions

Equation (8) is useful in achieving closure but can be temporarily disregarded. Of the remaining equations we find that Eq. (10) yields by itself the distribution of the mean flux $\overline{\rho v''c''}$ and that Eq. (6) gives subsequently the intensity $\overline{\rho v''^2}$. Thus to achieve closure we must deal with the third-order correlations $\overline{\rho u''v''c''}$ and $\overline{\rho u''v''^2}$; with the two dissipation terms, $\bar{\chi}_{vv}$ and $\bar{\chi}_{vc}$; and with the velocity-chemistry term, $\overline{v''\omega}$.

As indicated earlier, the modeling of the third-order correlations is facilitated by consideration of the conditioned pdf's. We temporarily switch to subscript notation so that both the x- and y-velocity components are treated compactly.

‡This independence depends on neglect of the redistributive effects of pressure fluctuations, a neglect we argue is appropriate for the thin reaction zones of interest here. A reviewer has raised the interesting question of whether this independence extends to conventionally averaged quantities. To examine this question would require consideration of the counterpart of Eqs. (1-10) using conventional averaging; the large number of extra terms arising from the decomposition of the density into mean and fluctuating components appears to make resolution of this question uncertain.

§Although the right side of Eq. (3) is zero, the first term on the left side describes the mean momentum in the y direction while the second describes the mean force in that direction, the force referred to here.

The conditioned mean velocity components within reactants and products can be written as

$$\bar u_{ir}(x) = \int du_i u_i P(u_i, 0; x) \quad \bar u_{ip}(x) = \int du_i u_i P(u_i, 1; x) \tag{13}$$

The corresponding conditioned intensities are likewise given by

$$\overline{u'^2_{ir}}(x) = \int du_i u_i^2 P(u_i, 0; x) - \bar u^2_{ir}(x)$$
$$\overline{u'^2_{ip}}(x) = \int du_i u_i^2 P(u_i, 1; x) - \bar u^2_{ip}(x) \tag{14}$$

Finally, we have conditioned correlations given by

$$\overline{u_i' u_{jr}'}(x) = \int du_i u_i \int du_j u_j P(u_i, u_j, 0; x) - \bar u_{ir} \bar u_{jr}(x)$$
$$\overline{u_i' u_{jp}'}(x) = \int du_i u_i \int du_j u_j P(u_i, u_j, 1; x) - \bar u_{ip} \bar u_{jp}(x) \tag{15}$$

We note that the conditioned pdf's $P(u_i, u_j, 0; x)$ and $P(u_i, u_j, 1; x)$ can be measured by a modest extension of the technique used in Refs. 6-8.

For $\gamma \ll 1$ the unconditioned quantities appearing in the conservation equations are expressible in terms of conditioned statistics. Of interest in the present study are the following examples

$$\tilde u_i = (1-\bar c)\bar u_{ir} + \bar c \bar u_{ip} + O(\gamma)$$

$$\overline{\rho u_i'' u_j''}/\bar\rho = \bar c(1-\bar c)(\bar u_{ip} - \bar u_{ir})(\bar u_{jp} - \bar u_{jr}) + (1-\bar c)\overline{u_i' u_{jr}'} + \bar c\, \overline{u_i' u_{jp}'} + O(\gamma)$$

$$\overline{\rho u_i'' c''}/\bar\rho = \bar c(1-\bar c)(\bar u_{ip} - \bar u_{ir}) + O(\gamma)$$

$$\overline{\rho u''^2_i}/ = \bar c(1-\bar c)(\bar u_{ip} - \bar u_{ir})^2 + (1-\bar c)\overline{u'^2_{ir}} + \bar c\overline{u'^2_{ip}} + O(\gamma)$$

$$\overline{\rho u_i'' u_j'' c''}/\bar\rho = \bar c(1-\bar c)[(\bar u_{ip} - \tilde u_i)(\bar u_{jp} - \tilde u_j) - (\bar u_{ir} - \tilde u_i)(\bar u_{jr} - \tilde u_j) + \overline{u_i' u_{jp}'} - \overline{u_i' u_{jr}'}] + O(\gamma)$$

$$\overline{\rho u''^2_i u_j''}/\bar\rho = (1-\bar c)[(\bar u_{ir} - \tilde u_i)^2(\bar u_{jr} - \tilde u_j) + \overline{u'^2_{ir}}(\bar u_{jr} - \tilde u_j) + 2\overline{u_i' u_{jr}'}(\bar u_{ir} - \tilde u_i) + \overline{u'^2_i u_{jr}'}] + \bar c[(\bar u_{ip} - \tilde u_i)^2(\bar u_{jp} - \tilde u_j) + \overline{u'^2_{ip}}(\bar u_{jp} - \tilde u_j) + 2\overline{u_i' u_{jp}'}(\bar u_{ip} - \tilde u_i) + \overline{u'^2_i u_{jp}'}] + O(\gamma) \tag{16}$$

Several comments regarding Eqs. (16) are indicated. It is interesting to note that the unconditioned quantities of second or higher order in the velocity components, e.g., $\overline{\rho u''^2}$, involve contributions from the bimodality appearing as the difference between mean values in reactants and products and from fluctuations within reactants and products. We find that except near the edges of the reaction zone where $\bar c = 0$, 1 the unconditioned quantities are dominated by the first contribution. Equations such as Eqs. (16) have important implications for experiments in which the thermochemical state is described by the Bray-Moss model of premixed combustion. For example, they show that measurements of conditioned statistics of two orthogonal velocity components permit determination of a wide range of unconditioned statistics.¶ These equations appear to provide the basis for extension of the modeling used here and in Refs. 2 and 3 for the treatment of simple planar flows to the more general case of flows which are two-dimensional in the mean. Finally, we note that gradient transport legislates that some of the unconditioned quantities in Eqs. (16) be zero. For example, if $\overline{\rho u_i'' c''}$ corresponds to $\overline{\rho v'' c''}$, the mean flux of product in the tangential direction, a quantity of interest in the present study, we would have according to gradient transport $\overline{\rho v'' c''} \propto -\partial\tilde c/\partial y = 0$. This implies that $\bar v_r = \bar v_p$; we effectively assess the validity of such a statement by avoiding either this assumption or other constraints on the conditioned tangential velocity components. In due course we find that $\bar v_r \neq \bar v_p$ and thus again demonstrate the inapplicability of gradient transport in variable density turbulence. A further consequence of this finding is that the mean streamlines of reactants and products are deflected within the reaction zone and are not coincident with the mean unconditioned streamline.

¶If the conditioned pdf's may be considered as Gaussian, measurements of conditioned mean and conditioned intensities are all that are required for the determination of all the statistics.

Closing the Equations

Our analysis is facilitated if we make assumptions concerning the conditioned statistics within reactants. We first make the benign assumption that the turbulence approaching the flame is isotropic so that $\overline{u'^2_0} = \overline{v'^2_0} = \overline{w'^2_0} = (2/3)\tilde q_0$ and $\overline{u'v_0'} = \overline{u'^2 v_0'} = \overline{u'v'^2_0} = 0$. Now we argue that as long as the parcels of reactants survive within the reaction zone they are only subjected to pressure stresses since viscous stresses are confined to their boundaries where chemical reaction takes place. As a consequence we assume $\overline{u'v_r'} = \overline{u'^2 v_r'} = \overline{u'v'^2_r} = 0$. It, of course, would be valuable to have experimental results testing this assumption.

With these general considerations we can take up the modeling specifically needed for Eq. (10). From Eqs. (16) we have with our assumption of $\overline{u'v_r'} = 0$ taken into account and with the indicators $O(\gamma)$ dropped

$$\tilde v = (1-\bar c)\bar v_r + \bar c \bar v_p = \hat\tau \tilde u/\tau$$

$$\overline{\rho v'' c''}/\bar\rho = \bar c(1-\bar c)(\bar v_p - \bar v_r)$$

$$\overline{\rho u'' v''}/\bar\rho = \bar c(1-\bar c)(\bar u_p - \bar u_r)(\bar v_p - \bar v_r) + \bar c\overline{u'v_p'}$$

$$= -\hat\tau \tilde u_0^2 \bar c(1+\tau\bar c)\overline{\rho u'' v'' c''}/\bar\rho = \bar c(1-\bar c)[(\bar u_p - \tilde u)(\bar v_p - \tilde v) - (\bar u_r - \tilde u)(\bar v_r - \tilde v) + \overline{u'v_p'}] \tag{17}$$

The first two of these equations permit $\bar v_r$ and $\bar v_p$ to be expressed in terms of $\tilde u$ and $\overline{\rho v'' c''}$ while the third permits the conditioned stress $\overline{u'v_p'}$ to be eliminated from the final equation and, thus, the mean flux in Eq. (10) to be expressed in terms of prime dependent variables. This sequence of steps is indicative of the closure methodology achieved by use of the conditioned pdf's in the Bray-Moss-Libby model.

The dissipation term $\tilde\chi_{vc}$ is given as in Refs. 2 and 3 by consideration of the laminar flamelet model of reacting surfaces.[10] Thus, we take

$$\tilde\chi_{vc} = \kappa_{vc}\bar\omega(\bar v_p - \bar v_r) \tag{18}$$

where κ_{vc} is an empirical constant.**

Finally, the velocity-chemistry term $\overline{v''\omega}$ in Eq. (10) can be calculated as in Ref. 2 on the basis of an estimate for the interior distribution $f(u,c;x)$ and for the rate of production $\omega(c)$.[11] We obtain

$$\overline{v''\omega} = \bar\omega(\phi_n - \bar c)(\bar v_p - \bar v_r) \tag{19}$$

**Because the present analysis leads to several additional empirical constants we find it convenient to alter the notation of Refs. 2 and 3. It is worth noting that the presence of $\bar\omega$ in Eq. (18) implies that Eq. (4), the conservation equation for the mean product, must be incorporated in the final equations.

where $\phi_n = n/(n+1)$ and $n>2$ is an integer exponent in the expression for $w(c)$, taken here to be 5.††

The treatment of Eq. (6) is not as straightforward. Although as suggested earlier, consistent application of the Bray-Moss-Libby model to the conservation equations for the higher-order correlations appears to result in a closed, expanded set of differential equations, present purposes are achieved by a simpler formulation calling for two ad hoc models for the conditioned statistics of the fluctuations of the v-velocity component. To see the need for such models consider the following equations representing specializations of Eqs. (16):

$$\overline{\rho v''^2}/\bar{\rho} = \tilde{c}(1-\tilde{c})(\bar{v}_p - \bar{v}_r)^2 + (1-\tilde{c})\overline{v'^2_r} + \tilde{c}\overline{v'^2_p}$$

$$\begin{aligned}\overline{\rho u'' v''^2}/\bar{\rho} = {} & (1-\tilde{c})[(v_r - \tilde{v})^2(\bar{u}_r - \tilde{u}) + \overline{v'^2_r}(\bar{u}_r - \tilde{u})] \\ & + \tilde{c}[(\bar{v}_p - \tilde{v})^2(\bar{u}_p - \tilde{u}) + 2\overline{u'v'_p}(\bar{v}_p - \tilde{v}) \\ & + \overline{v'^2_p}(\bar{u}_p - \tilde{u}) + \overline{u'v'^2_p}] \end{aligned} \quad (20)$$

These equations involve three new conditioned quantities: $\overline{v'^2_r}$, $\overline{v'^2_p}$, and $\overline{u'v'^2_p}$. If the mean flux $\overline{\rho u'' v''^2}$ is to be eliminated, it is clear we require two additional equations. In formulating these equations it is important to recall that the conditioned intensities and correlations are significant only at the edges of the reaction zone but are dominated elsewhere by quantities given deterministically. Thus, simple models are adequate for present purposes.

In Ref. 3 an equation relating the conditioned intensities within reactants and products is used. Here we adopt that equation and write

$$[(\overline{v'^2_p} - \overline{v'^2_r})/(\bar{v}_p - \bar{v}_r)^2] = \lambda_v \quad (21)$$

where λ_v is an empirical constant. The basis for Eq. (21) is that we expect the differences in the conditioned intensities are related to the differences in the conditioned velocities. For the second equation we assume the plausible relation

$$\frac{\overline{u'v'^2_p}}{\overline{u'^2v'_p}} = \left[\frac{\overline{v'^2_p}}{\overline{u'^2_p}}\right]^{1/2} \quad (22)$$

To use Eq. (22) we must eliminate the conditioned correlation $\overline{u'^2v'_p}$. This can be done by returning to Eq. (8), the conservation equation for the Reynolds shear stress. Equation (8) describes the unconditioned flux $\overline{\rho u''^2v''}$, which from Eqs. (16) and our earlier considerations, becomes

$$\begin{aligned}\overline{\rho u''^2v''}/\bar{\rho} = {} & (1-\tilde{c})[(\bar{u}_r - \tilde{u})^2(\bar{v}_r - \tilde{v}) + \overline{u'^2_r}(\bar{v}_r - \tilde{v})] \\ & + \tilde{c}[(\bar{u}_p - \tilde{u})^2(\bar{v}_p - \tilde{v}) + 2\overline{u'v'_p}(\bar{u}_p - \tilde{u}) \\ & + \overline{u'^2_p}(\bar{v}_p - \tilde{v}) + \overline{u'^2v'_p}] \end{aligned} \quad (23)$$

Thus, we see that by adding Eq. (8) as an auxiliary differential equation, an addition requiring only a model for the dissipation term arising therein, we can calculate $\overline{u'^2v'_p}$ and can achieve closure of Eq. (6). It is worth noting that the considerably greater difficulty encountered in closing this equation compared to that involved with Eq. (10) is associated with the third-order velocity correlation, $\overline{\rho u''v''^2}$; the second-order velocity correlation, $\overline{\rho u''v''c''}$, is considerably simpler to handle.

††Reference 12 recently provides an alternative calculation of $f(c;x)$ taking into account the strain that flamelets are subjected to in a turbulent reaction zone. The fluid mechanical model involves counterflowing reactants and products while the chemical reaction is described in terms of activation energy asymptotics. Such a calculation permits consideration of local extinction.

The arguments leading to Eq. (18) are applied to the dissipation terms in Eqs. (6) and (8) and result in

$$\tilde{\chi}_{vv} = \kappa_{vv}\bar{\omega}(\bar{v}_p - \bar{v}_r)^2 \quad \tilde{\chi}_{uv} = \kappa_{uv}\bar{\omega}(\bar{u}_p - \bar{u}_r)(\bar{v}_p - \bar{v}_r) \quad (24)$$

where κ_{vv} and κ_{uv} are our final empirical constants.

In the interest of brevity we do not discuss in detail the modeling necessary to close Eqs. (2) and (9). However, it is noted that the terms $\overline{u''}$ and $\overline{c''}$ multiplying the pressure gradient in these equations are determined without modeling for the Bray-Moss thermochemical system as

$$\overline{u''} = \tau\overline{\rho u''c''}/\rho_0 \quad \overline{c''} = \tau\overline{\rho c''^2}/\rho_0 = \tau\tilde{c}(1-\tilde{c})/(1+\tau\tilde{c}) + O(\gamma)$$

Final Equations and Their Solutions

As in our earlier work, numerical treatment is facilitated by letting $\tilde{c}$ be the independent variable and by treating the resultant singular points at the end of the 0, 1 range by appropriate regular expansions. For completeness we give the equations for the normal flame but with slightly modified notation to accommodate the new dependent variables required in the present analysis. We define the following dimensionless variables:

$$\begin{aligned} & I_u = \overline{\rho u''^2}/\rho_0\tilde{u}_0^2 \quad F_u = \overline{\rho u''c''}/\rho_0\tilde{u}_0 \quad L_{uu} = \overline{\rho u''^3}/\rho_0\tilde{u}_0^3 \\ & K_u = \overline{\rho u''^2c''}/\rho_0\tilde{u}_0^2 \quad I_v = \overline{\rho v''^2}/\rho_0\tilde{u}_0^2 \quad F_v = \overline{\rho v''c''}/\rho_0\tilde{u}_0 \\ & L_{vv} = \overline{\rho u''v''^2}/\rho_0\tilde{u}_0^3 \quad K_v = \overline{\rho u''v''c''}/\rho_0\tilde{u}_0^2 \\ & L_{uv} = \overline{\rho u''^2v''}/\rho_0\tilde{u}_0^3 \end{aligned} \quad (25)$$

In terms of these variables and with $\tilde{c}$ as the independent variable we obtain from Eqs. (5), (9), (10), (6), and (8) written in order and with the modeling described earlier the following equations:

$$\begin{aligned} & [(1+\tau\tilde{c})I_u]' + L'_{uu} + 2\tau I_u = 2\tau F_u(\tau + I'_u) \\ & \quad - \kappa_{uu}(1+F'_u)(hF_u)^2 \end{aligned} \quad (26)$$

$$\begin{aligned} & [(1+\tau\tilde{c})F_u]' + K'_u + \tau F_u + I_u = \tau(\tau + I'_u)/h \\ & \quad - (\tilde{c} - \phi_n)(1+F'_u)hF_u - \kappa_{uc}(1+F'_u)hF_u \end{aligned} \quad (27)$$

$$\begin{aligned} & [(1+\tau\tilde{c})F_v]' + K'_v + \hat{\tau}F_u - \hat{\tau}\tilde{c} = -(\tilde{c} - \phi_n)(1+F'_u)hF_v \\ & \quad - \kappa_{vc}(1+F'_u)hF_v \end{aligned} \quad (28)$$

$$[(1+\tau\tilde{c})I_v]' + L'_{vv} = 2\hat{\tau}^2\tilde{c} - \kappa_{vv}(1+F'_u)(hF_v)^2 \quad (29)$$

$$\begin{aligned} & -[(1+\tau\tilde{c})\tilde{c}\hat{\tau}]' + L'_{uv} + \hat{\tau}I_u = \tau F_v(\tau + I'_u) \\ & \quad - \kappa_{uv}(1+F'_u)h^2F_uF_v \end{aligned} \quad (30)$$

where the primes denote differentiation with respect to $\tilde{c}$, where $h = h(\tilde{c}) \equiv (1+\tau\tilde{c})/[\tilde{c}(1-\tilde{c})]$ is introduced for notational convenience, and where κ_{uu} and κ_{uc} are, respectively, κ_2 and κ_1 of Refs. 2 and 3.‡‡

The various flux terms in these equations are expressed in terms of prime variables according to the following equations:

$$L_{uu} = (1 - 2\tilde{c} + 3\lambda_u)h^2F_u^3 \quad K_u = (1 - 2\tilde{c} + \lambda_u)hF_u^2$$

$$K_v = -\hat{\tau}\tilde{c}(1-\tilde{c}) - \tilde{c}hF_uF_v$$

‡‡Note that the factor $(1+F'_u)$ appearing repeatedly in these equations is proportional to $\bar{\omega}$ and arises because of the incorporation of Eq. (4) into the final equations.

$$L_{vv} = \{L_{uv} + [(1-\lambda_u)h^2F_uF_v + 2(1+\tau\tilde{c})\hat{\tau}]F_u\}$$
$$\times\left[\frac{(1+\tau\tilde{c})I_v - (\tilde{c}+\lambda_v)(1-\tilde{c})h^2F_v^2}{(1+\tau\tilde{c})I_u - (\tilde{c}+\lambda_u)(1-\tilde{c})h^2F_u^2}\right]^{1/2}$$
$$-[(1-\lambda_v)h^2F_uF_v + 2(1+\tau\tilde{c})\hat{\tau}]F_v \qquad (31)$$

Several comments concerning these final equations are indicated. We assume isotropy of the turbulence in the oncoming reactants so that at the cold edge of the reaction zone, i.e., at $\tilde{c}=0$, $I_0 = I_u(0) = u'^2_0/\tilde{u}_0^2 = I_v(0)$. At the hot edge, i.e., at $\tilde{c}=1$, $I_{u\infty} = I_u(1) = u'^2_\infty/[\tilde{u}_0^2(1+\tau)]$ and $I_{v\infty} = I_v(1) = v'^2_\infty/[\tilde{u}_0^2(1+\tau)]$. At both edges $F_u = L_u = K_u = F_v = K_v = 0$. At the cold edge $L_{uv} = L_{vv} = 0$ while its value at the hot edge is calculated. The intensity parameter I_0 is specified while those on the downstream side of the reaction zone, $I_{u\infty}$, $I_{v\infty}$ are calculated as part of the solution.

These equations involve two sets of empirical constants: κ_{uu}, κ_{uc}, κ_{vc}, and κ_{vv} and κ_{uv} arising from the dissipation terms; and λ_u and λ_v arising from the relation between the conditioned intensities of the velocity fluctuations. In the absence of appropriate experimental data relative to oblique flames we take values established for normal flames in Ref. 3 and assume they apply to the present case as well with the exception of λ_v. If we require that the magnitudes of the differences $(v'^2_p - v'^2_r)$ and $(u'^2_p - u'^2_r)$ be the same, we must take $\lambda_v = \lambda_u/\hat{\tau}^2$.§§ Thus, we let

$$\kappa_{uu} = 0.25 = \kappa_{vv} = \kappa_{uv}, \quad \kappa_{uc} = 0.85 = \kappa_{vc}, \quad \lambda_u = 0.1 = \lambda_v\hat{\tau}^2 \quad (32)$$

The parameter I_0, relating to the turbulent flame speed $\tilde{u}_0$, arises in the boundary conditions; we take the value $I_0 = 0.23$ which correlates a large amount of experimental data.[3]

We consider that there are two levels of uncertainty associated with the values of the parameters added by consideration of obliquity. To calculate the mean tangential flux and, thus, the conditioned tangential velocities requires only a value for κ_{vc} which can be reasonably associated with κ_{uc} selected in Ref. 3 by comparison with experimental results. We thus consider the calculation of these quantities to be relatively reliable. The determination of the intensity of the fluctuations of the v-velocity component involves the other new parameters, κ_{vv}, κ_{uv}, and λ_v. The need to deal with three parameters, each subject to uncertainty, makes these calculations less reliable. Nevertheless, we believe that the resultant predictions are qualitatively correct.

Appropriate expansions of the dependent variables are required in the neighborhood of $\tilde{c}=0$, 1 (again we change the notation of Refs. 2 and 3 to accommodate the additional parameters called for by the present analysis). As $\tilde{c}\sim 0$ we find the following expansions to be appropriate

$$I_u \sim I_0 + \beta_{u0}\tilde{c} \qquad F_u \sim \gamma_{u0}\tilde{c} \qquad F_v \sim \gamma_{v0}\tilde{c}$$
$$I_v \sim I_0 + \beta_{v0}\tilde{c} \qquad L_{uv} \sim \zeta_0\tilde{c} \qquad (33)$$

When these forms are substituted into Eqs. (26-30), there result five algebraic equations for the five multipliers of $\tilde{c}$. The equations for I_u and F_u lead to a quadratic equation for γ_{u0} and a linear equation for β_{u0} if γ_{u0} is known. The equation for F_v gives γ_{v0} and the equations for I_v and L_{uv} give β_{v0} and ζ_0, all via linear algebraic equations. Thus Eqs. (33) provide appropriate initial values for starting a numerical integration in the neighborhood of $\tilde{c}=0$, typically at $\tilde{c}=0.005$.

§§This form must be modified by replacing $\tan\theta$ with $\sin\theta$ if $\theta\sim 90$ deg. Since our principal interest is in the small flame angles found experimentally,[4] this modification is not carried out except in connection with Fig. 6 where we wish to show the behavior as $\theta\sim 90$ deg.

The corresponding expansions applicable to $\tilde{c}\sim 1$ are more complex since there are acceptable solutions involving arbitrary constants. Direct extensions to the present case of the analysis of Refs. 2 and 3 lead to the following forms.

$$I_u \sim I_{u\infty} + \beta_{u1}(1-\tilde{c}) + B_uC_u(1-\tilde{c})^{\alpha_u}$$
$$F_u \sim \gamma_{u1}(1-\tilde{c}) + C_u(1-\tilde{c})^{\alpha_u} \quad F_v \sim C_v(1-\tilde{c})^{\alpha_v}$$
$$I_v \sim I_{v\infty} + \beta_{v1}(1-\tilde{c}) + B_vC_u(1-\tilde{c})^{\alpha_u} + D_vC_v(1-\tilde{c})^{\alpha_v}$$
$$L_{uv} \sim L_{uv\infty} + \zeta_1(1-\tilde{c}) + D_LC_v(1-\tilde{c})^{\alpha_v} \qquad (34)$$

In this case substitution into Eqs. (26-30) and collection of constant terms and of terms proportional to $(1-\tilde{c})^{\alpha_u-1}$ and $(1-\tilde{c})^{\alpha_v-1}$ yield algebraic equations for the multipliers of $(1-\tilde{c})$; for the exponents α_u and α_v; and for the quantities B_u, B_v, D_v, and D_L. The coefficients C_u and C_v are arbitrary but can be selected to make the solutions continuous to $\tilde{c}=1$. Equations (34) are used to continue to $\tilde{c}=1$ the numerical solution initiated in the neighborhood of $\tilde{c}=0$. Thus, the quantities on the left side and the value of $\tilde{c}$ on the right side are known. The sequence for the determination of the various coefficients in Eqs. (34) follows closely that described in the discussion of Eqs. (33).

Before taking up other issues, it is worth considering the production terms in the two intensity equations, Eqs. (5) and (6) in primitive form and Eqs. (26) and (29) in final form. The intensity $\overline{\rho u''^2}$ is affected by two such terms; one, associated with dilatation and proportional to $d\tilde{u}/dx$, diminishes the intensity. A second, due to the pressure gradient, increases the intensity provided, as indeed is the case of the mean flux of product $\overline{\rho u''c''}>0$. In contrast, the intensity $\overline{\rho v''^2}$ is affected by only one production term, that associated with the Reynolds shear stress. In final form this term is proportional to $\hat{\tau}^2$ and increases the intensity. If the invariance of the third intensity in the form $\overline{\rho w''^2}/\bar{\rho}$ is recalled, it is appreciated that the balance of turbulent kinetic energy in oblique flames involves all three of the mechanisms discussed earlier.

Conditioned Streamlines

We recall our earlier comment that the conditioned tangential velocities, $\bar{v}_r$ and $\bar{v}_p$, are unconstrained by a priori assumptions. If F_v is obtained from the solution of Eq. (28), we can compute these velocities a posteriori from the first two of Eqs. (17), i.e., from the following equations.

$$\frac{\bar{v}_r}{\tilde{u}_0} = \frac{1+\tau\tilde{c}}{\tan\theta} - \tilde{c}hF_v \qquad \frac{\bar{v}_p}{\tilde{u}_0} = \frac{1+\tau\tilde{c}}{\tan\theta} + (1-\tilde{c})hF_v \qquad (35)$$

Similar relations prevail for the conditioned normal velocities, $\bar{u}_r$ and $\bar{u}_p$, so that the conditioned mean flow deflections, ψ_r and ψ_p, can be calculated from the relations $\tan\psi_r = \bar{v}_r/\bar{u}_r$ and $\tan\psi_p = \bar{v}_p/\bar{u}_p$, respectively. Distributions of these angles can be used to indicate the streamlines within reactants and products. Conditioned flow deflections greater than the complement of the flame angle θ denoted ψ_0 (see Fig. 1) imply that the associated streamline is deflected toward the y axis relative to the mean unconditioned streamline while deflections less than ψ_0 imply deflection toward the x axis. The distribution of ψ_r from the cold to the hot edge of the reaction zone indicates the streamline of parcels of reactants which survive to the latter edge. The corresponding distribution of ψ_p does likewise for the streamline of parcels of products which are created at the former edge. These two streamlines represent limiting behavior. The path lines of parcels of reactants that are randomly destroyed within the reaction zone are continued as path lines of product. It is clear that the random path lines are bounded by the limiting streamlines. These considerations and the implications for future experiments given thereby are interesting aspects of the present study.

It is worth noting that the conditioned flow deflections depend on both conditioned velocity components. Thus even if $\bar{v}_r = \bar{v}_p$ (the case when $F_v \equiv 0$), in general $\psi_r \neq \psi_p$ because $\bar{u}_r \neq \bar{u}_p$. It is also worth noting that these predictions relative to the conditioned streamlines rely on computation of the two mean flux parameters, F_u and F_v. The accuracy of the former is established by the results of Ref. 3 while the calculation of the second involves a straightforward extension of those results and is therefore considered sufficiently accurate for these predictions to be viewed with confidence.

III. Numerical Results

The numerical analysis is, in principal, carried out sequentially; the solutions of Eqs. (26) and (27) are independent of the flame angle θ and can be considered given by Ref. 3. The mean flux in the tangential direction is next obtained by solution of Eq. (28). Finally, Eqs. (29) and (30) yield the intensity of the velocity fluctuations associated with I_v. All solutions are obtained by integration, from the neighborhood of $\tilde{c}=0$ where Eqs. (33) apply to the neighborhood of $\tilde{c}=1$ where Eqs. (34) apply. At a value in the latter neighborhood, typically $\tilde{c}=0.999$, the algebraic equations yield $I_{u\infty}$ and $I_{v\infty}$. Solutions are identified by the values of τ and θ. However, we find that the functional dependence on θ can be incorporated into the dependent variables so that the extent of the heat release contained in τ is the principal parameter.

Valid solutions are subject to certain restrictions which limit somewhat the values of the empirical constants given in Eqs. (32).¶¶ On physical grounds the intensities of the velocity fluctuations, both unconditioned and conditioned, and the mean chemical production must be non-negative. The behavior of the solutions at the end points $\tilde{c}=0$, 1 is restricted on mathematical grounds, e.g., the values of some of the coefficients are given by polynomials which must have acceptable solutions and the exponents α_u, α_v must be between unity and two. All the solutions we present respect these restrictions.

Mean Flux in the Tangential Direction

We need not discuss in detail the solutions for I_u and F_u since they are given in Ref. 3. Consider the solutions for the mean flux of product in the tangential direction in terms of the quantity F_v. Examination of Eq. (28) indicates that $F_v = \hat{\tau}\hat{F}_v$ where $\hat{F}_v$ is independent of the flame angle. Thus, we can convert Eq. (28) to give $\hat{F}_v$ with τ as the sole parameter. The important implication from this finding is that the tangential flux increases as $\tan\theta$ decreases and vanishes for $\tau=0$. Thus, the present theory is consistent with gradient transport for constant density turbulence and predicts the alteration of the turbulent exchange in the tangential direction due to density variations.

In Fig. 2 we show for $\tau=6.5$, the value corresponding to the experiments of Moss,[6] the distribution of F_u and $\hat{F}_v$. We see that for small flame angles the tangential flux tends to be considerably greater than the flux in the normal direction, although that tendency is compromised by increases in the extent of heat release.

Figure 3 shows the distribution of $\hat{F}_v$ with $\tilde{c}$ for three different degrees of heat release. The mean flux F_v is positive throughout the reaction zone and increases monotonically with τ.

Examination of Fig. 3 exposes an interesting, special solution of Eq. (28) which determines $\hat{F}_v(\tilde{c})$. If, instead of the values $\phi_n = 0.833$, $\kappa_{vc} = 0.85$ used in obtaining the results given in Fig. 3, we have $\phi_n = \kappa_{vc}$, it is easy to show that Eq. (28) and the boundary conditions applicable to F_v are satisfied by $h\hat{F}_v = 1$, a remarkably simple solution. Equating ϕ_n and κ_{vc}

¶¶See Ref. 3 for a discussion of these restrictions as they arise in normal flames. Similar arguments pertain to the present study.

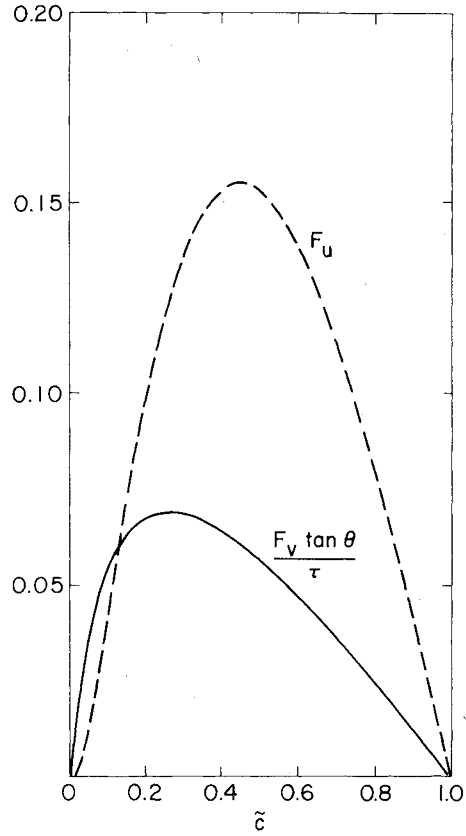

Fig. 2 Comparison of the normal and tangential mean flux of product, $\tau = 6.5$.

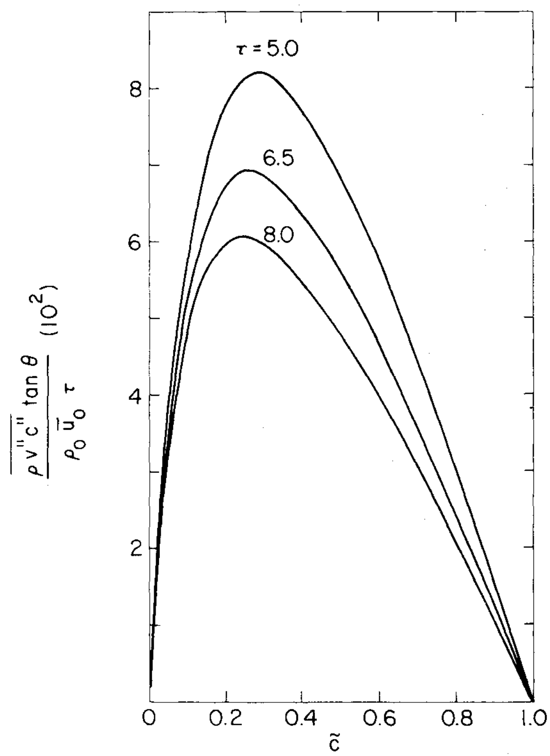

Fig. 3 Distributions of the nondimensionalized mean flux of product in the tangential direction.

implies that part of the product correlation $\overline{\rho v'' c''}$ due to velocity-reaction coupling as reflected in the term $\overline{v'' \omega}$ is cancelled by molecular dissipation. When this cancellation prevails, the other influences on the correlation are balanced in such a fashion that the simple solution prevails. The results in Fig. 3 are close to those described by this solution.

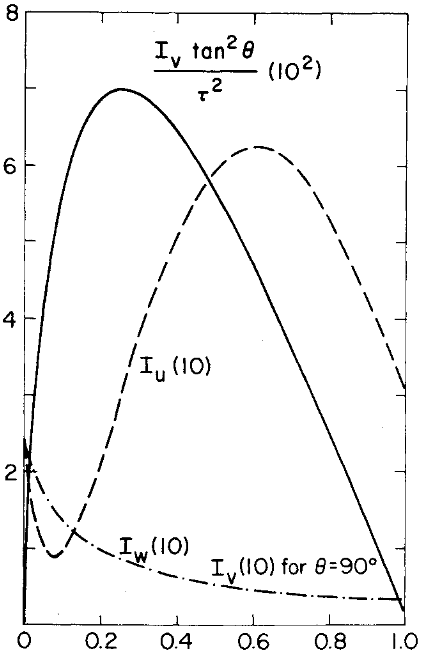

Fig. 4 Comparison of the three intensity parameters, $\tau=6.5$.

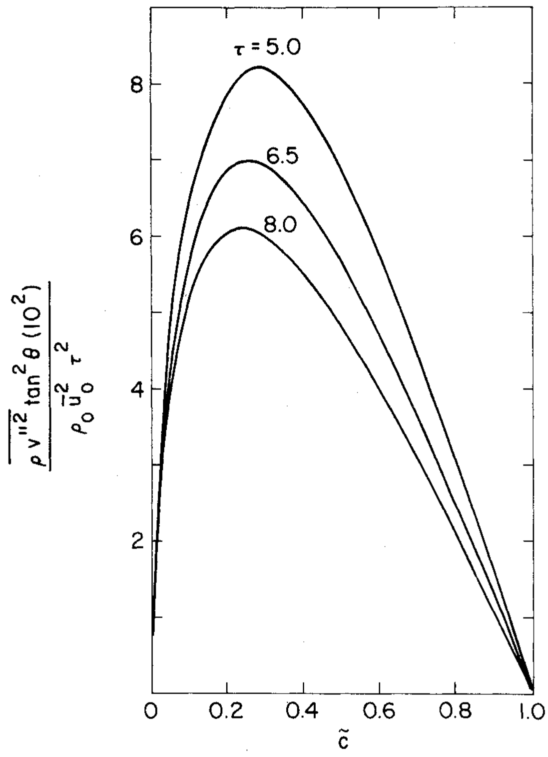

Fig. 5 Distribution of the intensity of the fluctuations of the v-velocity component.

Intensity of the Velocity Fluctuations

As might be expected from our earlier observations the suppression of the functional dependence of I_v on the flame angle cannot be obtained from analytic considerations. However, we find numerically that for $\theta<60$ deg the quantity $I_v/\hat{\tau}^2$ is insensitive to θ. Accordingly, we give the results obtained in terms thereof.

Figure 4 shows for $\tau=6.5$ the distribution of the three intensity parameters, I_u, I_v, and $I_w=\overline{\rho w''^2}/\rho_0\tilde{u}_0^2$. Two distributions related to I_v are shown; one corresponds to a normal flame for which $I_v=I_w=I_0/(1+\tau\tilde{c})$. A second corresponds to oblique flames with $\theta<60$ deg. The different character of the distributions of I_u and I_v reflects the existence of counterbalancing production terms in the equation describing the former, and of only one such term in that for the latter. Again we see the tendency of the intensity related to I_v to increase with decreasing flame angle, a tendency which is compromised by the extent of the heat release.

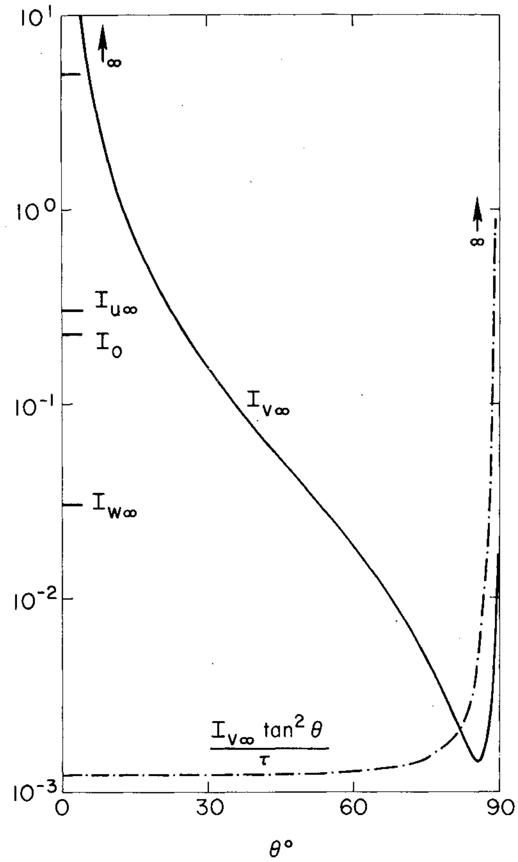

Fig. 6 Variation of the intensity parameters downstream of oblique reaction zones, $\tau=6.5$.

We show in Fig. 5 the distributions of I_v with $\tilde{c}$ for various values of τ. The results are calculated for $\theta=7.5$ deg but apply for any angle $\theta<60$ deg. Although there is an apparent decrease in the maximum intensity with increasing τ, the presence of τ in the denominator of the ordinate results in an increase in intensity with increasing τ as expected on physical grounds.

With the intensity parameter I_v available it is worth showing in Fig. 6 the variation of the intensities downstream of the reaction zone with flame angle for $\tau=6.5$. The results relative to $I_{v\infty}$ are given in terms of the nearly similar form, namely, in terms of $I_{v\infty}/\hat{\tau}^2$, and directly in terms of $I_{v\infty}$. Note that our assertion regarding insensitivity of the former form for $\theta<60$ deg is confirmed. For small values of θ we see that the turbulent kinetic energy downstream of the flame is determined by $I_{v\infty}$, $I_{u\infty}$, and $I_{w\infty}$ in that order of importance. For example, with $\theta=7.5$ deg it is predicted that $I_{v\infty}=3$, $I_{u\infty}=0.3$ and $I_{w\infty}=0.03$ and, thus, that the turbulent fluctuations are nearly two dimensional in the x-y plane. It is this anisotropy which is subject to the redistributive effect of pressure fluctuations in the region downstream of the reaction zone.

Conditioned Flow Deflections

We now consider the distributions of the flow deflections across the reaction zone, both unconditioned and conditioned. We rewrite Eqs. (35) in the following illuminating

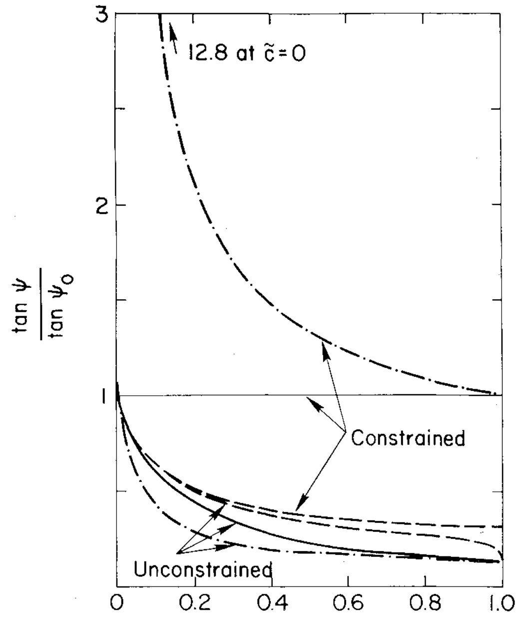

Fig. 7 Distributions of conditioned flow deflections, $\tau=6.5$; —— unconditioned; ——— reactants; — · — · — products.

form

$$\frac{\tan\psi_r}{\tan\psi_0}=\frac{1-\tau\hat{F}_v/(1-\tilde{c})}{1-F_u/(1-\tilde{c})}\quad \frac{\tan\psi_p}{\tan\psi_0}=\frac{1+\tau\hat{F}_v/\tilde{c}}{1+F_u/\tilde{c}} \tag{36}$$

The right sides of Eqs. (36) are independent of flame angle θ so that it is possible to show in Fig. 7 their variations across the reaction zone, again for $\tau=6.5$. It is seen that the streamline deflection of the reactants is everywhere less than ψ_0. Thus the limiting streamline for reactants is on the x-axis side of the undeflected, unconditioned streamline. On the contrary, the limiting streamline for the products is everywhere on the y-axis side of the undeflected streamline. The physical explanation for this behavior resides in the differential effect on the heavy reactants and light products of the tangential force field associated with the Reynolds shear stresses. This effect is analogous to that prevailing in the normal direction due to the mean pressure gradient.

The results in Fig. 7 indicate that the differences in the two sets of conditioned velocities, those in reactants and products in the two coordinate directions, are sufficient to encourage their experimental determination. Thus, extension of the measurements of one velocity component by Moss[6] and Shephard and Moss[7] so that two orthogonal velocities are obtained is suggested.

It is interesting to contrast these results with those that prevail in the case of flames with unconstrained streamlines and, thus, no Reynolds shear stresses. The equations for the unconditioned deflection angle ψ and for the two conditioned deflections such as those given by Eqs. (36) are readily found to be

$$\frac{\tan\psi}{\tan\psi_0}=\frac{1}{1+\tau\tilde{c}}\quad \frac{\tan\psi_r}{\tan\psi_0}=\frac{1}{(1+\tau\tilde{c})[1-F_u/(1-\tilde{c})]}$$

$$\frac{\tan\psi_p}{\tan\psi_0}=\frac{1}{(1+\tau\tilde{c})(1+F_u/\tilde{c})} \tag{37}$$

We show on Fig. 7 the distributions of the right sides of these equations. Since the difference in the conditioned deflections is due in this case solely to the differences in the x-wise velocity components, they are considerably smaller than in the case of a constrained mean streamline.

Several comments relative to these results are indicated. Suppose that an extension of the optical technique used by Moss[6] and Shephard and Moss[7] is used to obtain the distributions through the reaction zone of both sets of conditioned velocities, $\bar{u}_r$, $\bar{v}_r$, $\bar{u}_p$, and $\bar{v}_p$. From such measurements the distributions of ψ_r and ψ_r vs $\tilde{c}$ can be determined and compared with the limiting predictions of Eqs. (36) and (37). We expect that in open flames the results will be close to those of Eqs. (37) and will imply relatively unconstrained mean streamlines and the absence of significant Reynolds shear stresses. In ducted flows we expect such results to be closer to those given by Eqs. (36), i.e., to those associated with constrained streamlines with significant Reynolds shear stresses. Note that in such an experiment simultaneous measurements of two velocity components are not required. If simultaneous data are available, the accuracy of the general relations given by Eqs. (16) and of the assumption of negligible conditioned stresses within reactants $\overline{u'v'_r}$ can be assessed. Note that this assessment would be valuable whether the mean streamlines are undeflected or otherwise.

IV. Conclusions

We have extended an earlier nongradient theory of premixed turbulent flames that are free of turbulent shear stresses to describe flames oblique to the oncoming reactants and having such stresses. Thus, all three mechanisms influencing the balance of turbulent kinetic energy are operative. It is found from an examination of the primative conservation equations that for oblique flames which are infinite and planar, the intensity of the fluctuations of the velocity component normal to the flame and the mean flux of product in that direction are unaffected by obliquity. Thus, our earlier findings of countergradient diffusion and production of turbulence by interaction of density inhomogeneities and the mean pressure drop across the flame prevail in oblique flames.

The extension focuses on calculation of the mean flux of product tangent to the flame and calculation of the intensity of the fluctuations of the velocity component in that direction. From the former calculation it is possible to determine the conditioned velocity components tangent to the flame and the conditioned flow deflections, i.e., mean flow directions within reactants and products. It is found that the limiting streamlines of parcels of these two fluids are distinct and significantly different from the mean streamline which is assumed undeflected within the reaction zone. Since this calculation involves a modest, straightforward extension of our earlier results, supported by comparison with experiment, we consider these predictions to be accurate. The differences in the conditioned flow deflections are such as to encourage their experimental investigation by extension of the technique used by Moss[6] and Shephard and Moss.[7]

The calculation of the intensity of the fluctuations of the tangential velocity component involves a significant extension of our previous analysis and the addition of three empirical constants whose values are uncertain. The quantitative accuracy of the predictions of this intensity is, therefore, uncertain but we believe the results are qualitatively correct. The most interesting finding is that the turbulence downstream of an oblique flame is essentially two-dimensional in the x-y plane and involves a kinetic energy considerably greater than that in the oncoming reactants since two production mechanisms, the mean pressure drop across the flame and the Reynolds shear stresses, are operative.

The methodology for describing the velocities in oblique flames appears to provide a means for extending the analysis to premixed turbulent combustion with statistical characteristics varying with two spatial coordinates, provided the Bray-Moss model of the thermochemistry applies.

Acknowledgments

The research reported here was supported by the Office of Naval Research under Project SQUID. The authors gratefully acknowledge helpful comments by K.N.C. Bray and the contribution of Barbara Hanson in preparing this manuscript for publication.

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